Statistics Review

- PSY 450W
- Dr. Schuetze

Two central ways of using numbers.

1. **Descriptive Statistics:**
   - Simple quantitative description or summary.
     - Batting average in baseball
     - Grade-point average

2. **Inferential Statistics:**
   - Conduct analyses on samples
     - Compare groups (experimental v. control...)
   - Use statistical operations to generalize the results to a population.

Describing data

We characterize the general trend or character of data using two key statistics:

1. **Central tendency** or general “drift” of the scores.
   - Mode → most common score
   - Median → middle of the distribution
   - Mean → average score

2. **Variability** how diverse the scores are (how much vary from each other).
   - Range → …from the highest to lowest score
   - Standard deviation → “average” amount the scores vary from the Mean score
Mode

- Most frequent score in the distribution
- Example: scores = 16, 20, 21, 20, 36, 15, 25, 15, 12

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>% of cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>33</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
<td>11</td>
</tr>
</tbody>
</table>
- 15 is most common = mode

Mode

- Characteristics
  - Used for all numerical scales, particularly nominal.
  - Insensitive to extreme values or range of scores.
  - Unstable: sensitive to small shifts in number of cases.

Median

- Mid-point of a distribution of scores
- List scores in numerical order
  - 12, 15, 15, 15, 20, 20, 21, 25, 36
- Locate the score in the center of the sample
  - 12, 15, 15, 15, 20, 20, 21, 25, 36
  - The middle (5th out of 9) score = 20.
### Median

- Characteristics:
  - Sensitive to the range of scores
  - More stable than the mode
  - Not sensitive to extreme scores (e.g., changing highest score (36) to 100 would not change the median.

### Mean ($M$)

- The “average” score in a sample
- Most common measure of central tendency
  - Total all scores: $12 + 15 + 20 + 21 + 20 + 36 + 15 + 25 + 15 = 179$
  - Divide by “n” of scores: $179 / 9 = 19.9$

### Mean

- Characteristics:
  - Good for Ratio or interval scales
  - Sensitive to all observed values
  - Highly stable; with larger $n$ is insensitive to subtle changes in values
  - Can be highly sensitive to extreme values (particularly in smaller samples).
For a normal distribution, the mean, mode, and median are all same -- the center of the distribution.

Most variables in nature (and science) are normally distributed.

Age is a good example of a variable that is normally distributed.

Scores for age from a large community sample form a largely symmetrical distribution.

The Mean, Median, and mode are similar.

Any measure of central tendency well represents the data.
A skewed distribution has extreme scores in one direction. The extreme scores make the median higher than the mean. (The high scores to the right move the 50% point that direction...).

The mean gets pulled even higher. (Adding in some very high scores raises the average...).

Common examples:
- Behaviors such as alcohol or drug use:
  - Most people use none or moderate
  - A diminishing number use higher levels
- Demographic variables such as income

### Positive skew example
Example of typical strong positive skew; Drug & alcohol use (Community survey sample)

### Measures of Variability
- Variability: amount of fluctuation in data.
- 20, 30, 40, 50, 60, 70, 80
- 47, 48, 49, 50, 51, 52, 53
Measures of Variability

- **Range**: Difference between highest and lowest scores.
- **Variance**: Deviation from the mean of the scores. How much scores are spread out or dispersed around mean.
- **Standard Deviation**: Squared root of variance.

Variance: Standard Deviation

Estimates of Variance:

2. The **Standard deviation** ($S$) of scores around the Mean

- Similar to the “average” amount that each score deviates from the $M$ of the sample.
- “Standardizes” scores to a normal curve, allowing basic statistics to be used.
- More accurate & detailed than range:
  - A few extremely high or low scores (“outliers”) may make the range inaccurate.
  - $S$ assesses the deviation of all scores in the sample from the mean

Comparing Scores: Variance & Standard Deviation

The data sets have the same $M$, but differ in how widely their scores vary (their variance).

- **High variance**
  - $S = 2.4$

- **Less variance**
  - $S = 1.15$
Scales of Measurement

- Nominal Scale: observations are labeled and categorized (qualitative).
- Ordinal Scale: observations are ranked in terms of size/magnitude they are in relation to each other (qualitative).
- Interval Scale: equal differences (intervals) between numbers on the scale reflect equal differences in magnitude (quantitative).
- Ratio Scale: ratios of numbers do reflect ratios of magnitude (quantitative).

Normal Distribution

- Characteristics
  - Symmetrical
  - Three measures of central tendency are same value
  - Most scores fall close to mean
- Parametric Statistics: inferential stats used to analyze normally distributed interval/ratio scores.
- Nonparametric Statistics: inferential statistics used to analyze interval/ratio scores not normally distributed.

Testing Hypotheses

- Statistical Hypothesis: restatement of research hypothesis into two different hypotheses.
  - Alternative Hypothesis: statistical term for research hypothesis (H$_1$).
  - Null Hypothesis: Predicted relationship does not exist in the population (H$_0$).
null hypothesis. all scores differ from the \( M \) by chance alone.

statistical question (alternate hypothesis):
- does this score differ from the \( M \) by > chance?

using the normal distribution
- more extreme scores have a lower probability of occurring by chance alone
- the # of standard deviation units (‘z’ score) = the % of cases above or below the observed score (its “extremity”)

“statistical significance”

statistical significance
✓ by convention, we assume that a score with less than 5% probability of occurring [i.e., higher or lower than 95% of the other scores… \( p < .05 \)] has not occurred by chance alone.
✓ \( p < .05 \) corresponds to \( Z = 1.98 \); \( Z \) tells us if we can consider the effect (the distance from the \( M \)) to be “statistically significant.”
✓ if \( Z > 1.98 \) we consider the score to be “significantly” different from the mean

statistical significance & areas under the normal curve
95% of scores are between \( Z = -1.98 \) and \( Z = +1.98 \).

\( Z > +1.98 \) or \( Z < -1.98 \) has \( p < .05 \); a.k.a. it occurs by chance less than 5% of the time…

\( Z = -1.98 \)
\( Z = +1.98 \)

2.5% of cases

about 95% of cases
### One-tailed vs. Two-tailed

- Nondirectional hypothesis → two-tailed test
- Directional hypothesis → one-tailed test

### Errors in Hypothesis Testing

- **Type I Error**: the null hypothesis has been mistakenly rejected when it is actually true.
- **Type II Error**: the null hypothesis has been mistakenly accepted when it is actually false.

### Chi Square

- Nonparametric test: determines whether the frequencies of responses in our sample represent frequencies expected in the population.
- Contingency table
- Compares obtained frequencies with expected frequencies
<table>
<thead>
<tr>
<th></th>
<th>Compiled</th>
<th>Refused</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>6 (15.5)</td>
<td>74 (64.5)</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>(7.5)</td>
<td>(9.3)</td>
<td></td>
</tr>
<tr>
<td>Alone</td>
<td>25 (15.5)</td>
<td>55 (64.5)</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>(31.3)</td>
<td>(68.75)</td>
<td></td>
</tr>
<tr>
<td>Column Totals</td>
<td>31</td>
<td>129</td>
<td>160</td>
</tr>
</tbody>
</table>