This contradiction completes the proof of the theorem. Hence, in particular, $c_k(\lambda_k - \lambda_1) = 0$. We noted above that $c_k \neq 0$. Thus $\lambda_k = \lambda_1$. We conclude that not all of the given eigenvalues are distinct.

Because $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{k-1}$ are linearly independent, we have that $c_k \neq 0$.