# MAT 430 Review Problems for Midterm on Chapters 1, 2, and 3.1-3.3.3.
# Midterm – Friday, March 20, 2020.

**Problem 1.** State the subset axiom.

**Problem 2.** Let $\mathcal{F}$ be a set and let $C \in \mathcal{F}$. Prove $\bigcap \mathcal{F} \subseteq C$.

**Problem 3.** Let $\mathcal{F}$ and $A$ be sets. Prove that if $A \subseteq C$ for some $C \in \mathcal{F}$, then $A \subseteq \bigcup \mathcal{F}$.

**Problem 4.** Let $\mathcal{F}$ and $A$ be sets and $\mathcal{F} \neq \emptyset$. Prove that if $A \subseteq C$ for all $C \in \mathcal{F}$, then $A \subseteq \bigcap \mathcal{F}$.

**Problem 5.** Let $\mathcal{F}$ and $A$ be sets. Suppose $C \subseteq A$ for every $C \in \mathcal{F}$. Prove that $\bigcup \mathcal{F} \subseteq A$.

**Problem 6.** Let $A$ be a set. Prove that $\bigcup \mathcal{P}(A) = A$.

**Problem 7.** Let $A$ be a set. Prove that $A \subseteq \mathcal{P}\left(\bigcup A\right)$.

**Problem 8.** Prove that $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$.

**Problem 9.** Let $\mathcal{F}$ and $\mathcal{G}$ be sets. Then there is a unique set $\mathcal{D}$ such that for all $Y$, we have that $Y \in \mathcal{D}$ if and only if $Y = A \cup B$ for some $A \in \mathcal{F}$ and some $B \in \mathcal{G}$.

**Problem 10.** Let $F$ be a relation from $X$ to $Y$. Suppose $A \subseteq X$ and $B \subseteq X$. Prove that $F[A] \setminus F[B] \subseteq F[A \setminus B]$.

**Problem 11.** Let $R$ be a relation. We know by the discussion in the text prior to Definition 3.2.5 that $\operatorname{dom}(R)$ and $\operatorname{ran}(R)$ are sets. Let $T$ be a relation. Evaluate the set $\operatorname{dom}(T) \cap \operatorname{ran}(T^{-1})$.

**Problem 12.** Let $R$ be a relation. Prove that $\operatorname{fld}(R) = \bigcup\bigcup R$. (See Definition 3.2.5)

**Problem 13.** Let $F$ and $G$ be relations, where $\operatorname{dom}(F) \subseteq X$ and $A \subseteq X$. Prove that $F \restriction A$, $F[A]$, and $F \circ G$ are sets.

**Problem 14.** Let $R$ be a relation on $A$. Prove that $R$ is transitive if and only if $R \circ R \subseteq R$.

**Problem 15.** Let $\mathcal{G}$ be a nonempty set of transitive relations on $A$. Prove that $\bigcap \mathcal{G}$ is transitive.

**Problem 16.** Let $R$ be an equivalence relation on $A$. Prove that $R \circ R = R$.

**Problem 17.** Let $R$ and $S$ be single-rooted relations. Prove that $R \circ S$ is single-rooted.

**Problem 18.** Let $F\colon X \to Y$ be onto $Y$, $C \subseteq Y$, $D \subseteq Y$. Prove $F^{-1}[C] \subseteq F^{-1}[D]$ implies $C \subseteq D$.

**Problem 19.** Let $\mathcal{H}$ be a set of functions. Suppose that for all $f$ and $g$ in $\mathcal{H}$ we have either $f \subseteq g$ or $g \subseteq f$. Prove that $\bigcup \mathcal{H}$ is a function.

**Problem 20.** Assume $f\colon X \to Y$ is onto $Y$. Let $C \subseteq Y$. Prove that $f[f^{-1}[C]] = C$.

**Problem 21.** Let $F$ and $G$ be functions, and let $A \subseteq \operatorname{dom}(G)$. Prove that $(F \circ G)[A] = F[G[A]]$.

**Problem 22.** Let $F\colon A \to B$ be a onto $B$. Let $G\colon \mathcal{P}(A) \to \mathcal{P}(B)$ be the function defined by $G(X) = F[X]$ for each $X \in \mathcal{P}(A)$. Prove that $G$ is onto $\mathcal{P}(B)$.

**Problem 23.** Prove $\bigcap\{\mathcal{P}(B) : B \in \mathcal{B}\} = \mathcal{P}(\bigcap \mathcal{B})$.

**Problem 24.** Prove $\bigcup\{\mathcal{P}(B) : B \in \mathcal{B}\} \subseteq \mathcal{P}(\bigcup \mathcal{B})$.

**Problem 25.** Let $F$ be a function and let $A \subseteq B \subseteq \operatorname{dom}(F)$. Prove that $F[A] \subseteq F[B]$.

**Problem 26.** Let $g\colon A \to B$ and $f\colon B \to C$. Suppose that $(f \circ g)\colon A \to C$ is one-to-one. Prove that $g$ is one-to-one.

**Problem 27.** Let $g\colon A \to B$ and $f\colon B \to C$. Suppose that $(f \circ g)\colon A \to C$ is one-to-one and $g$ is onto. Prove that $f$ is one-to-one.