MAT 300 Review Problems for Chapter 3 and Sections 4.2, 4.4
Exam #2 on Friday, November 8, 2019

A proof by mathematical induction must NOT have the notations $P(1)$, $P(n)$, or $P(n+1)$ appearing anywhere in the proof.

Prove[1] the following theorems:

1. **Theorem.** Let x and y be real numbers. Then $(x-y)(x^2+xy+y^2) = x^3 - y^3$.
2. **Theorem.** Let a and b be real numbers. If $a < 0$ and $b < 0$, then $(a+b)^2 > a^2 + b^2$.
3. **Theorem.** Let $n \geq 2$ be a natural number. If $2^n > n$, then $2^{n+1} > n+1$.
4. **Theorem.** For all real numbers $y > 0$, there is a real number $x < 0$ such that $y^2+2xy = -x^2$.
5. **Theorem.** Let a and b are real numbers such that $a > 0$ and $b < -4$. Then $ab+b < -4(a+1)$.
6. **Theorem.** Suppose n is an integer. Then $15 \mid n$ if and only if $3 \mid n$ and $5 \mid n$.
7. **Theorem.** Let m, a, b, c, d be integers where $m > 1$. If $m \mid (a-b)$ and $m \mid (c-d)$, then $m \mid ((a+c)-(b+d))$.
8. **Theorem.** Let a, b, d be in $\mathbb{R}$. If $0 \leq a < d$ and $0 \leq b < d$, then $a - b < d$ and $b - a < d$.
9. **Theorem.** For all integers a, b, and c, if $c \mid a$ and $c \mid b$, then $c \mid (a-b)$.
10. **Theorem.** For every $x \in \mathbb{R}$, there is a real number y such that $yx^2 - 3x = -2y$.
11. **Theorem.** There is a $y \in \mathbb{R}$, such that $yx + 6 = 2x + 3y$ for all real numbers x.
12. **Theorem.** For every natural number $n \geq 1$, $2 + 6 + 18 + \cdots + 2 \cdot 3^{n-1} = 3^n - 1$.
13. **Theorem.** Let x and y be positive real numbers. Then $\frac{x+y}{2} \geq \sqrt{xy}$.
14. **Theorem.** The real number $2 + \frac{1}{2}\sqrt{2}$ is irrational.
15. **Theorem.** For every natural number $n \geq 1$, we have $\sum_{k=1}^{n} \frac{1}{4k^2-1} = \frac{n}{2n+1}$.
16. **Theorem.** $\sum_{k=1}^{n} k \cdot k! = (n+1)! - 1$, for all natural numbers $n \geq 1$.[2]
17. **Theorem.** $8 \mid (9^n - 1)$, for every integer $n \geq 1$.
18. **Theorem.** Let $x \geq -1$ be a real number. Then $(1+x)^n \geq 1 + nx$ for all integers $n \geq 1$.
19. **Theorem.** For every integer $n \geq 1$, $\sum_{k=1}^{n} 2 \cdot 3^{k-1} = 3^n - 1$.

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[1] On the exam, to receive full credit, your proofs must be clearly composed, logically correct, and readable.

[2] Recall that $(k+1)!(k+2) = (k+2)!$.