MAT 300 Review Problems for Exam #1 on Chapters 1 and 2.
Examination #1 – Friday, September 27, 2019.

Problem 1. Using propositional logical laws, show that the $(P \to R) \vee (Q \to R) \Leftrightarrow (P \wedge Q) \to R$.
[You must justify your steps.]

Problem 2. Using propositional logic laws, show $\neg(P \vee \neg Q) \vee (\neg P \wedge \neg Q) \Leftrightarrow \neg P$.
[You must justify your steps.]

Problem 3. Determine whether the sentences are true or false in the universe $\mathbb{R}$.

1. $(\forall x > 2)(x < 4 \to x^2 < 16)$.
2. $(\forall x < 2)(x < 4 \to x^2 < 16)$.
3. $(\exists x < 2)(x < 4 \wedge x^2 < 4)$.
4. $(\exists x > 2)(x < 4 \wedge x^2 < 4)$.

Problem 4. Determine whether the sentences are true or false.

1. $(\forall x \in \mathbb{N})(x > 2 \to 3x < 2^x)$.
2. $(\forall x \in \mathbb{N})(x > 4 \to 3x < 2^x)$.
3. $(\exists x \in \mathbb{Z})(\frac{1}{5+x} \in \mathbb{N})$.
4. $(\exists x \in \mathbb{N})(\frac{1}{5+x} \in \mathbb{Z})$.

Problem 5. Evaluate the truth sets:

1. $\{x \in \mathbb{R} : (\exists y \in \mathbb{R})(x = y^2)\}$.
2. $\{x \in \mathbb{R} : (\forall y \in \mathbb{R})(x < y^2)\}$.
3. $\{x \in \mathbb{R} : (\forall y > 2)(x < y^2 + 1)\}$.
4. $\{x \in \mathbb{R} : (\exists y > 2)(x < y^2 + 1)\}$.
5. $\{x \in \mathbb{Z} : (\exists y \in \mathbb{Z})(x = y^2)\}$.
6. $\{w \in \mathbb{Z} : (\exists x \in \mathbb{Z})(w = 3x)\}$.
7. $\{q \in \mathbb{Q} : (\exists x \in \mathbb{Q}^+)(qx = 1)\}$.
8. $\{q \in \mathbb{Q} : (\forall x \in \mathbb{Q})(qx = x)\}$.

Problem 6. Show, using logical laws, that the propositional sentences

- $(P \to Q) \wedge (P \to R)$
- $P \to (Q \wedge R)$

are logically equivalent. [You must justify your steps.]

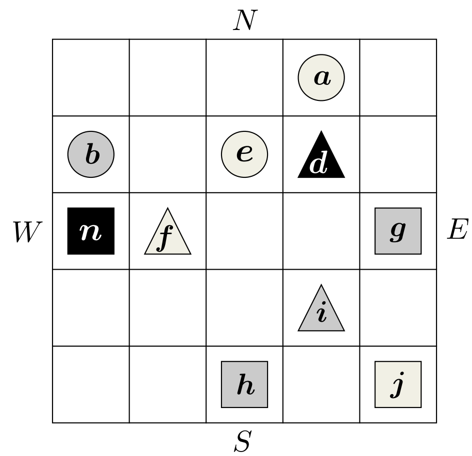
Problem 7. Using quantifier negation laws and propositional logic laws, express each of the following statements in a logically equivalent form that does not contain $\neg$. The universe is the set of real numbers.

1. $\neg(\forall x > 2)(\exists y < 2)(x < 4 \to xy < 16)$.
2. $\neg(\exists x > 2)(\forall y < 2)(x < 4 \to xy < 16)$.
3. $\neg(\forall x \in \mathbb{N})(\exists y \in \mathbb{Z})(x > 2 \to x < y)$.
4. $\neg(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})(x < y)$.

Problem 8. Using quantifier negation laws and propositional logic laws, express each of the following statements in a logically equivalent form that does not contain $\neg$. The universe is the set of real numbers and a and b are constant real numbers.

1. $\neg(\forall \varepsilon > 0)(\exists \delta > 0)\forall x(|x - a| < \delta \to \left|x^2 - b\right| < \varepsilon)$.
2. $\neg(\forall \varepsilon > 0)(\exists N \in \mathbb{N})(\forall n \in \mathbb{N})(n > N \to \left|\frac{1}{n} - a\right| < \varepsilon)$.

Problem 9. Consider the following Tarskian World.



Determine the truth or falsity of each of the following statements. The universe consists of all the objects in the above Tarski world. **You do NOT have to justify your answer.**

1. $\forall x(S(x) \to (G(x) \vee I(x)))$. Circle one: True False
2. $\forall y(T(y) \to N(y, i))$. Circle one: True False
3. $\forall y(T(y) \to \exists x N(y, x))$. Circle one: True False
4. $\forall x(S(x) \to \exists y(C(y) \wedge K(x, y)))$. Circle one: True False
5. $\exists y(S(y) \wedge \forall x(C(x) \to \neg K(x, y)))$. Circle one: True False

Problem 10. Using the Tarskian predicates, translate the following English sentences into logical sentences.

1. Every gray square is north of some triangle.
2. Some circle is west of every square.
3. Some circle is north of a white triangle.
4. All squares are the same color as some triangle.
5. All black squares are west of all gray circles.
6. No square has the same color as any circle.

Problem 11. Some of the arguments below are valid by universal modus ponens or modus tollens; others are invalid. State which are valid and which are invalid.

1. All healthy people eat an apple a day.
   Johnny is not a healthy person.
   $\therefore$ Johnny does not eat an apple a day.
2. All healthy people eat an apple a day.
   Johnny eats an apple a day.
   $\therefore$ Johnny is a healthy person.
3. All freshmen must take writing.
   Dan is a freshman.
   $\therefore$ Dan must take writing.
4. All natural numbers are integers.
   π is not an integer.
   $\therefore$ π is not a natural number.
5. All integers are natural numbers.
   -5 is an integer.
   $\therefore$ -5 is a natural number.