5.1 Basic Definitions of Set Theory

(b)  $(-2,4) \cup (-\infty,2)$ . (c)  $(-\infty,0] \setminus (-\infty,2]$ . (d)  $\mathbb{R} \setminus (2,\infty)$ . (e)  $(\mathbb{R} \setminus (-\infty,2]) \cup (1,\infty)$ .

(2.) Express the following sets as truth sets.

- (a)  $A = \{1, 4, 9, 16, 25, \dots\}$
- (b)  $B = \{\dots, -15, -10, -5, 0, 5, 10, 15, \dots\}.$

**3.** Evaluate the truth sets.

- (a)  $A = \{x \in \mathbb{N} : 0 < x^2 < 24\}$
- (b)  $B = \{y \in \mathbb{Z} : y \mid 12\}$
- (c)  $C = \{z \in \mathbb{N} : 4 \mid z\}$
- (d)  $D = \{y \in \mathbb{R}^- : 1 \le y^2 \le 4\}.$
- **4.** Let *A*, *B*, and *C* be the sets in Exercise 3. Evaluate the following sets:  $A \cup B$ ,  $A \cap C$ ,  $A \setminus B$ ,  $B \setminus A$ , and  $C \setminus (A \cup B)$ .
- **5.** Find two elements in the set  $\mathbb{R} \setminus \mathbb{Q}$ . Explain why  $\mathbb{Q} \setminus \mathbb{R} = \emptyset$ .
- 6. Let  $A = \{2,3\}$  and  $B = \{a,b,c\}$ . Evaluate  $A \times A$ ,  $A \times B$ ,  $B \times A$ , and  $B \times B$ .
- 7. Let  $A = \{2,3\}$  and  $B = \{3,a\}$ . Evaluate  $\mathcal{P}(A \cup B)$  and  $\mathcal{P}(A) \cup \mathcal{P}(B)$ .
- **8.** Find  $\mathcal{P}(\emptyset)$  and  $\mathcal{P}(\mathcal{P}(\emptyset))$ .
- **9.** Let  $A = \{2,3\}, B = \{a,b\}$  and  $C = \{x,y\}$ . Evaluate  $(A \times B) \times C$  and  $\mathcal{P}(A \times B)$ .
- **10.** Let  $A = \{2,3\}, B = \{3,4\}$  and  $C = \{3,y\}$ . Is  $A \times (B \cup C) = (A \times B) \cup (A \times C)$ ?
- Let *A*, *B*, and *C* be sets. Determine which of the following statements are always true and which are not always true.
  - (a) If  $x \in A$ , then  $x \in A \cup B$ .
  - (b) If  $x \in A \cup B$ , then  $x \in A$ .
  - (c) If  $x \in B$  and  $A \subseteq B$ , then  $x \in A$ .
  - (d) If  $x \notin B$  and  $A \subseteq B$ , then  $x \notin A$ .
  - (e) If  $x \in A$  and  $A \not\subseteq B$ , then  $x \notin B$ .
  - (f) If  $x \in C$  and A = C, then  $x \in A$ .
  - (g) If  $x \in A \cap B$ , then  $x \in A \cup B$ .
  - (h) If  $x \notin A \cap B$ , then  $x \notin A \cup B$ .
  - (i) If  $x \notin A \setminus B$ , then  $x \notin A$  or  $x \in B$ .
  - (j) If  $(x, y) \in A \times B$ , then  $x \in A$  and  $y \in B$ .
  - (k) If  $(x, y) \notin A \times B$ , then  $y \in A$  and  $x \in B$ .
  - (1) If  $A \in \mathcal{P}(B)$ , then  $A \subseteq B$ .
- 12. Let *E* be the set of even integers and let *O* be the set of odd integers. Is  $\{E, O\}$  a partition of  $\mathbb{Z}$ ? Justify your answer.
- **13.** Find a partition  $P = \{S_0, S_1, S_2, S_3\}$  of  $\mathbb{Z}$ , similar to the one in Example 7 on page 148, that breaks  $\mathbb{Z}$  up into 4 disjoint subsets.
- 14. Find a partition  $P = \{S_0, S_1, S_2, S_3, S_4\}$  of  $\mathbb{Z}$ , similar to the one in Example 7, that breaks  $\mathbb{Z}$  up into 5 disjoint subsets.