2. **Theorem.** Let n and k be integers where k is odd. Then n is odd if and only if $n = 2i + k$ for some integer i.
3. **Theorem.** Let n be an integer. Then $3 \mid n$ if and only if $3 \mid (5n+6)$.
4. **Theorem.** Let n be an integer. Then $15 \mid n$ if and only if $3 \mid n$ and $5 \mid n$.
5. **Theorem.** Let x be a real number. Then $|x| = 0$ if and only if $x = 0$.
6. **Theorem.** Let $c > 0$. For $x \in \mathbb{R}$, $|x| = c$ if and only if $x = -c$ or $x = c$.
7. **Theorem.** Let $c > 0$. For all real numbers x, $|x| < c$ if and only if $-c < x < c$.

Exercise Notes: For Exercise 2, in your proof of the direction ($\Rightarrow$) use the algebraic identity $n = n - k + k$. For Exercise 3, in your proof of the direction ($\Leftarrow$) use the algebraic identity $n = 6n - 5n$. For Exercise 4, the proof is very similar to the proof of Theorem 3.7.2. Note that $n = 6n - 5n$. For Exercises 6 and 7, in your proof of both directions ($\Leftarrow$) and ($\Rightarrow$) there are two cases: (1) $x \geq 0$ and (2) $x < 0$.

3.8 Indirect Proof

We are now familiar with "direct proofs" of mathematical statements. A direct proof establishes that a statement is true by using the definitions and previous results to logically derive the statement. An indirect proof of a statement can take two different forms: proof by contraposition and proof by contradiction. Proof by contraposition establishes the truth of an alternative statement whose truth implies the truth of the original statement. Proof by contradiction argues that the original statement cannot possibly be false and therefore, it must be true. In mathematics, indirect proofs are very common. The first indirect proof that we investigate is proof by contraposition, and then we shall pursue proof by contradiction.

3.8.1 Proof by Contraposition

There may be times when it is not easy to prove a conditional statement, say $\psi \to \varphi$; that is, using the assumption ψ it may be difficult to prove φ. In this case, logic can come to the rescue. Since $\psi \to \varphi$ and its contrapositive $\neg\varphi \to \neg\psi$ are logically equivalent, we can prove the contrapositive instead. Consequently, we can assume $\neg\varphi$ and try to prove $\neg\psi$. This alternative approach is called *proof by contraposition*.

Proof Strategy 3.8.1. Given a diagram containing the form

Prove $P \to Q$

to apply proof by contraposition, replace this form with

Assume $\neg Q$
    Prove $\neg P$.