Suppose that a statement of the form $P \to Q$ is an **assumption** and not the conclusion you are trying to prove. How can you **use** this assumption in your proof? Recall the inference rules modus ponens and modus tollens discussed in Section 1.3.3.

Assumption Strategy 3.3.4. Given a diagram containing the form

Assume $P \to Q$

there are two approaches:

(a) If you are assuming or can prove P, then you can conclude Q by modus ponens.
(b) If you are assuming or can prove $\neg Q$, then you can deduce $\neg P$ by modus tollens.

Exercises 3.3

1. Find a counterexample showing that the following conjecture is false: *Let a and b are real numbers. If $a < b$, then $a^2 < b^2$.*
2. Let x and y be real numbers satisfying $4x+5y \geq 6$. Prove if $x < 4$, then $y > -2$.
3. Suppose a and b are negative real numbers. Prove that if $a < b$, then $a^2 > b^2$.
4. Let a be a real number. Prove that if $a > 0$, then $(a+4)^2 > a^2 + 16$.
5. Let x be a real number. Prove that if $x \geq 4$, then $x^2 > 2x+1$.
6. Let n be an integer. Prove that if $n > 3$, then $2^{n-1} + 2^{n-2} + 2^{n-3} \leq 2^n$.
7. Prove that if $(1+x)^n \geq 1+nx$, then $(1+x)^{n+1} \geq 1+(n+1)x$, when $x > -1$ is a real number and n is a natural number.
8. Let $n \geq 2$ be an integer. Prove that if $2^n > n$, then $2^{n+1} > n+1$.
9. Show that Theorem 3.1.9 implies Theorem 3.3.2.

Exercise Notes: For Exercises 4–8, one should review the Substitution Properties of Inequality 3.3.3 and Example 1.

3.4 Statements of the Form $\forall x P(x)$ and $\exists x P(x)$

To prove that a statement is true for all x, we must prove that the statement is true for every element x in our universe. On the other hand, to prove that a statement is true for some x, we may have to first find such an individual and then prove that this individual satisfies the statement.