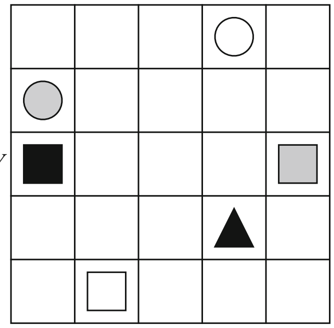

Fig. 2.5 A Tarskian World for Example 10

Example 10 Using the Tarskian predicates (as defined in Example 5 on page 49), determine the truth value of the following logical sentences in the Tarskian world in Fig. 2.5, below.

1. $\exists! x\, T(x)$.
2. $\exists! x\, C(x)$.
3. $\exists! x\,(G(x) \wedge C(x))$.
4. $\forall x \exists! y\, N(y,x)$.
5. $\exists! x \forall y\,(x \neq y \rightarrow W(y,x))$.
6. $\exists! x \forall y\,(x \neq y \rightarrow W(x,y))$.

Solution We first express each of the logical statements into English and then we will determine its truth value in the Tarskian world of Fig. 2.5.

1. $\exists! x\, T(x)$ means that "there is exactly one triangle." This is true!
2. $\exists! x\, C(x)$ states that "there is exactly one circle." This is false!
3. $\exists! x\,(G(x) \wedge C(x))$ declares that "there is exactly one grey circle," and this is true.
4. $\forall x \exists! y N(y,x)$ asserts that "for every individual x there is exactly one individual who is north of x." This is false in the given Tarskian world.
5. $\exists! x \forall y\,(x \neq y \rightarrow W(y,x))$ is translated to mean "there is exactly one individual x such that all the individuals who are different than x, are west of x." The grey square is this unique individual. So the statement is true.
6. $\exists! x \forall y\,(x \neq y \rightarrow W(x,y))$ is translated to mean "there is exactly one individual x such that all the individuals who are different than x, are east of x." The statement is false. Ⓢ

Exercises 2.4

1. Let the universe be a group of people and let $L(x,y)$ mean "x likes y." What do the following formulas mean in English?

(a) $\forall y \exists x L(x,y)$
(b) $\exists x \forall y L(x,y)$.

Show that these two statements are not logically equivalent by constructing a world, as in Example 3, where one statement is true while the other is false.

2. Write the following statement in logical form and then write a negation of this statement in English: *All even integers are twice some integer.* [Use the predicate $E(x)$ for "x is even," and let the universe be the set of integers.]

3. Determine whether the statements are true or false in the universe $\mathbb{R}$.

(a) $\forall a \exists x (x^2 = a)$.
(b) $\forall x \exists a (a + x = 0)$.
(c) $\exists a \forall x (a + x = 0)$.
(d) $\forall x \exists a (ax = 0)$.
(e) $\forall x \exists y (x < y)$.
(f) $\exists y \forall x (x < y)$.
(g) $\forall x \forall y (x = y \to x^2 = y^2)$.
(h) $\forall x \forall y (x^2 = y^2 \to x = y)$.

4. Using the given predicates, analyze the logical form of the following sentences.

(a) No one likes everyone. (Universe is a group of people.) [Let $L(x,y)$ mean "x likes y."]
(b) Someone likes no one. (Universe is a group of people.) [Let $L(x,y)$ mean "x likes y."]
(c) Every number is the cube of some number. (Universe is $\mathbb{R}$.)
(d) Someone in high school is smarter than everyone in college. (Universe is the set of all students.) [Let $H(x)$ mean "x is in high school," $C(x)$ mean "x is in college," and $S(x,y)$ mean "x is smarter than y."]

5. Using the Tarskian predicates given in Example 5, translate the following six English sentences into logical sentences.

(a) Every gray square is north of some triangle.
(b) Some circle is west of every square.
(c) Some circle is north of a white triangle.
(d) All squares are the same color as some triangle.
(e) All black squares are west of all gray circles.
(f) No square has the same color as any circle.

6. Using quantifier negation laws and propositional logic laws, express each of the following statements as a positive one. The universe is the set of real numbers.

(a) $\neg(\forall x > 2)(\exists y < 2)(x < 4 \to xy < 16)$.
(b) $\neg(\exists x > 2)(\forall y < 2)(x < 4 \to xy < 16)$.
(c) $\neg(\forall x \in \mathbb{N})(\exists y \in \mathbb{Z})(x > 2 \to x < y)$.
(d) $\neg(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})(x < y)$.