Exercises 2.1

1. Let $\frac{p}{q}$ and $\frac{r}{s}$ be rational numbers where p, q, r, s are integers and q, s are nonzero. Suppose $\frac{p}{q} = \frac{r}{s}$ and $p \neq 0$. Using Definition 2.1.3, show that $\frac{ps}{qs} = \frac{pr}{ps}$.

2. Let $R(x)$ be the predicate $x > \frac{1}{x}$. Indicate whether the statements $R(2)$, $R(-2)$, $R(\frac{1}{2})$, $R(-\frac{1}{2})$ are true or false. Now evaluate the following truth sets:

(a) $\{x \in \mathbb{R}^+ : x > \frac{1}{x}\}$.
(b) $\{x \in \mathbb{R}^- : x^2 > \frac{1}{x}\}$.
(c) $\{x \in \mathbb{Z} : x > \frac{1}{x} \text{ and } x > 2\}$.
(d) $\{x \in \mathbb{Z} : x > \frac{1}{x} \text{ and } x \not> 2)\}$.

3. Let $L(x,y)$ be the predicate "$x < y$." Determine if the following statements are true or false:

(a) $L(2,3) \to L(4,9)$.
(b) $L(-3,-2) \to L(9,4)$.
(c) $L(-2,-3) \to L(9,4)$.

4. Evaluate the truth sets:

(a) $\{x \in \mathbb{R} : x^2 < 9\}$.
(b) $\{x \in \mathbb{Z} : x^2 < 9\}$.
(c) $\{x \in \mathbb{R} : 2x + 9 \leq 5\}$.
(d) $\{x \in \mathbb{R} : x > 0 \text{ and } x^3 < \frac{16}{x}\}$.

5. Let $\frac{p}{q}$ and $\frac{r}{s}$ be rational numbers where p, q, r, s are integers and q, s are nonzero. Suppose $\frac{p}{q} = \frac{r}{s}$. Using Definition 2.1.3, show that $\frac{2p+q}{2q} = \frac{2r+s}{2s}$.

2.2 Quantifiers

Given a statement $P(x)$, which says something about the variable x, we want to express the fact that *every* element x in the universe makes $P(x)$ true. In addition, we may want to express the fact that *at least one* element x in the universe makes $P(x)$ true. To do this, we will form sentences using the quantifiers $\forall$ and $\exists$. The quantifier $\forall$ means "for all" and is called the *universal quantifier.* The quantifier $\exists$ means "there exists" and is called the *existential quantifier.* For example, we can form the sentences

1. $\forall x P(x)$ [means "for all x, $P(x)$"].
2. $\exists x P(x)$ [means "there exists an x such that $P(x)$"].