CHEAT SHEET

DeMorgan's Laws

1. $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
2. $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

Commutative Laws

1. $P \wedge Q \Leftrightarrow Q \wedge P$
2. $P \vee Q \Leftrightarrow Q \vee P$

Associative Laws

1. $P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$
2. $P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

Idempotent Laws

1. $P \wedge P \Leftrightarrow P$
2. $P \vee P \Leftrightarrow P$

Distribution Laws

1. $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
2. $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
3. $(Q \vee R) \wedge P \Leftrightarrow (Q \wedge P) \vee (R \wedge P)$
4. $(Q \wedge R) \vee P \Leftrightarrow (Q \vee P) \wedge (R \vee P)$

Double Negation Law

1. $\neg\neg P \Leftrightarrow P$

Tautology Law

1. $P \wedge (\text{a tautology}) \Leftrightarrow P$

Contradiction Law

1. $P \vee (\text{a contradiction}) \Leftrightarrow P$

Conditional Laws

1. $(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$
2. $(P \rightarrow Q) \Leftrightarrow \neg(P \wedge \neg Q)$
3. $\neg(P \rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

Contrapositive Law

1. $(P \rightarrow Q) \Leftrightarrow (\neg Q \rightarrow \neg P)$

Quantifier Negation Laws

1. $\neg\exists x P(x) \Leftrightarrow \forall x \neg P(x)$.
2. $\neg\forall x P(x) \Leftrightarrow \exists x \neg P(x)$.
3. $\neg(\forall x \in A)P(x) \Leftrightarrow (\exists x \in A)\neg P(x)$.
4. $\neg(\exists x \in A)P(x) \Leftrightarrow (\forall x \in A)\neg P(x)$.
5. $\neg(\forall x < c)P(x) \Leftrightarrow (\exists x < c)\neg P(x)$.
6. $\neg(\exists x < c)P(x) \Leftrightarrow (\forall x < c)\neg P(x)$.

Inference Rules

$\dfrac{P \to Q \quad P}{\therefore Q}$ (Modus Ponens)	$\dfrac{\forall x(P(x) \to Q(x)) \quad P(a)}{\therefore Q(a)}$ (Universal Modus Ponens)
$\dfrac{P \to Q \quad \neg Q}{\therefore \neg P}$ (Modus Tolens)	$\dfrac{\forall x(P(x) \to Q(x)) \quad \neg Q(a)}{\therefore \neg P(a)}$ (Universal Modus Tolens)

Tarskian Predicates

- $T(x)$ means "x is a triangle." $C(x)$ means "x is a circle." $S(x)$ means "x is a square."
- $I(x)$ means "x is white." $G(x)$ means "x is gray." $B(x)$ means "x is black."
- $N(x, y)$ means "x is on the northern side of y."
- $W(x, y)$ means "x is on the western side of y."
- $K(x, y)$ means "x has the same color as y."

Truth Tables

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	$\neg P$
T	F
F	T

Interval Notation

For real numbers a and b we have the following.

1. The open interval (a, b) is defined to be $(a, b) = \{x \in \mathbb{R} : a < x < b\}$.
2. The closed interval $[a, b]$ is defined to be $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$.
3. The half-open interval $(a, b]$ is defined to be $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$.
4. The half-open interval $[a, b)$ is defined to be $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$.
5. The interval (a, ∞) is defined to be $(a, \infty) = \{x \in \mathbb{R} : a < x\}$.
6. The interval $(-\infty, a)$ is defined to be $(-\infty, a) = \{x \in \mathbb{R} : x < a\}$.